MEASURING CREDIT RISK FOR SECURITIES MARGIN TRADING AND SECURITIES LENDING

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ABSTRACT. Taking fully account of the business rules and characteristics of Securities Margin Trading and Securities Lending (SMTSL) in Chinese market and risk aversion regulatory policy, this paper proposes an integrated model to measure credit risk (for customer default) in this business. Three key issues in the proposed approach are discussed: Threshold Breaking Probability (TBP), Default Probability (DP) and Loss Given Default (LGD). Among them, a Geometric Brownian Motion-based analytical approach and an empirical frequency mapping method are designed separately to determining the TBP, a formula integrating the TBP and the customer account information is developed to estimate the DP, and a Value-at-Risk (VaR)-based Monte Carlo Simulation algorithm and an empirical frequency mapping method are provided separately to computing the dynamic LGD of SMTSL. Applying the proposed models for the TBP, DP, and LGD, the credit economic capital can be calculated directly. Finally, a simple case study is provided to illustrate the model calculation.

Keywords: Credit Risk; Securities Margin Trading and Securities Lending; Geometric Brownian Motion; Dynamic LGD, Economic Capital

1. Brief Introduction. In view of the current rule for the business of Securities Margin Trading and Securities Lending (SMTSL) in China, the trading will be forcibly liquidated if the customer fails to deposit enough margin (collateral) to cover his/her short positions after receiving the margin call from the broker. In other words, the “forced liquidation” will be activated if the customer defaults. Hence, in this work, an event of “forced liquidation” can be regarded as an event of default. In order to measure the unexpected loss of credit risk (credit economic capital) of the SMTSL business, we need to calculate the Threshold Breaking Probability (TBP), Default Probability (DP), Loss given Default (LGD), and Default Correlation between the customer accounts.

2. Threshold Breaking Probability. The minimal maintenance margin ratio is stipulated as 130% by the regulator, the customer will receive margin call once the margin ratio of his/her credit accounts become lower than this ratio. Furthermore, if the customer fails to add sufficient margin within the time pre-specified, his/her account will then be forcibly liquidated. The default happens. Here, the margin ratio R(t) of the customer credit account is defined as the ratio of the value of collaterals
over the total liabilities value induced by the SMTSL trading, which can be calculated by the following formula:

$$R(t) = \frac{\text{cash} + \text{market value of total securities in customer credit account}}{\text{financing amount} + \text{market value of borrowed securities} + \text{interests and premium}}.$$ 

It is here clear that before we move to the default probability issues, we have to determine the probability that the margin ratio $R(t)$ of the customer credit account becomes lower than the minimal maintenance margin ratio (1.3) in certain horizon. This probability is exactly the Threshold Breaking Probability (TBP).

In order to model TBP in a rigorous manner, it is necessary to give some assumptions as well as notations as below:

**Assumptions:**

**A1:** The customers do not change their assets and liabilities of the credit accounts within the given horizon;

**A2:** The horizon $\Delta T$ is set to 0.5 years by considering the duration of SMTSL is regulated not longer than 0.5 years.

**Notations:**

- $t_0$: current time spot;
- $\Delta T$: time Horizon;
- $C_A(t_0 + \Delta T)$: customer’s own cash asset by the time $t_0 + \Delta T$;
- $\sum_i V_i^A(t_0 + \Delta T)$: total marginable securities of customer by the time $t_0 + \Delta T$;
- $C_{D,sell}(t_0)$: the amount of cash asset by selling the securities borrowed;
- $\sum_j V_j^{D,\text{buy}}(t_0 + \Delta T)$: the market value of the securities bought in by the customer by the time $t_0 + \Delta T$;
- $C_{D,\text{buy}}(t_0)$: financing amount by the customer;
- $\sum_k V_k^{D,\text{sell}}(t_0 + \Delta T)$: market value of the securities borrowed by the time $t_0 + \Delta T$;
- $P_m$: interests and premium.

With the above notation and based on the account information at time $t_0$, the threshold breaking probability (TBP) of the customer credit account over the duration

$$\text{TBP} = \Pr\left\{ \frac{C_A(t_0) + \sum_i V_i^A(t_0 + \Delta T) + C_{D,\text{sell}}(t_0) + \sum_j V_j^{D,\text{buy}}(t_0 + \Delta T)}{C_{D,\text{buy}}(t_0) + \sum_k V_k^{D,\text{sell}}(t_0 + \Delta T) + P_m} \leq 1.3 \right\}, \quad (1)$$

or equivalently,

$$\text{TBP} = \Pr\left\{ \sum_i V_i^A(t_0 + \Delta T) + \sum_j V_j^{D,\text{buy}}(t_0 + \Delta T) - 1.3 \sum_k V_k^{D,\text{sell}}(t_0 + \Delta T) \leq K \right\}, \quad (2)$$

where $K$ is a constant given by

$$K = 1.3(C_{D,\text{buy}}(t_0) + P_m) - C_A(t_0) - C_{D,\text{sell}}(t_0). \quad (3)$$

**2.1. Analytical Approach.** For the sake of a clearer explanation for this approach, we assume the following portfolio for the securities in the customer credit account:

$$P = \sum_{i=1}^{n} w_i V_i,$$

where $w_i$ is the quantity coefficient of the security $i$, which from (2) is the long position for the securities bought in, and is equal to “-1.3*short position” for the securities shorted, $V_i$ is the market price of the security $i$. 
Furthermore, in addition to the assumptions A1, A2, the analytical approach builds on the following key assumption:

**A3**: The security market price $V_i$ obeys the Geometric Brownian Motion:
\[
dV_i = \mu_i V_i dt + \sigma_i V_i dz,
\]
where $dz$ is the Standard Wiener Process.

**A4**: The increments $\Delta V_i$ of $V_i, i = 1, 2, \ldots$ have identical distribution.

The above A4 is a very common assumption employed frequently in financial risk management (for instance in the calculation of Value-at-Risk, see Hull, 2006), and from A4, we know that the increments of portfolio are also identically distributed.

On the basis of the above assumptions, we can obtain the distribution of the increments of portfolio in customer credit account in any duration $\Delta T$. In fact, given a small inter-arrival time $\Delta t$, it follows from the relation
\[
\Delta V_i = \mu_i V_i(t) \Delta t + \sigma_i V_i(t) \phi_i \sqrt{\Delta t}, \quad \phi_i \sim N(0,1)
\]
that the distribution of the increment $\Delta P_t$ by time $t$ of the portfolio $P$ can be determined as
\[
\Delta P_t = \Delta \left( \sum_{i=1}^{n} w_i V_i \right) \sim N \left( \Delta t \sum_{i=1}^{n} \mu_i V_i(t), \sqrt{\Delta t \sum_{i=1}^{n} \sum_{j} w_i w_j p_{ij} \sigma_i \sigma_j V_i(t) V_j(t)} \right), \quad (4)
\]
where $p_{ij}$ is the correlation between $\Delta V_i$ and $\Delta V_j$. Furthermore, denote
\[
\mu(\Delta P, t) = \Delta t \sum_{i=1}^{n} w_i \mu_i V_i(t), \quad \sigma(\Delta P, t) = \sqrt{\Delta t \sum_{i=1}^{n} \sum_{j} w_i w_j p_{ij} \sigma_i \sigma_j V_i(t) V_j(t)}, \quad (5)
\]
as the mean and standard deviation of $\Delta P_t$, respectively, then in the any duration $\Delta T = M \Delta t$, it follows from the identical distribution property of $\Delta P_t$ that
\[
\Delta P_T = \sum_{k=1}^{M} \Delta P_t(k) \sim N \left( M \mu(\Delta P, t_0), \sqrt{M} \sigma(\Delta P, t_0) \right), \quad (6)
\]
with $t_0$ being the current time spot.

Thus, the probability of threshold being violated in $\Delta T$ in (2) of the portfolio in credit account is equivalent to
\[
TBP = \Pr\{P(t_0 + \Delta T) \leq K\} = \Pr\{P(t_0) + \Delta P_T \leq K\},
\]
where $P(t)$ is the market value of the portfolio by the time $t$. It is easy to see that the above probability further reads
\[
\Pr\{\Delta P_T \leq K - P(t_0)\} = \Phi \left( \frac{K - P(t_0) - \Delta T \cdot \mu(\Delta P, t_0)}{\sqrt{\Delta T} \cdot \sigma(\Delta P, t_0)} \right), \quad (7)
\]
where $\Phi$ is the accumulate distribution function form normal distribution, $K$ is determined by (3), $\mu(\Delta P, t_0)$ and $\sigma(\Delta P, t_0)$ given by (5), and
\[
P(t_0) = \sum_{i} V_i^A(t_0) + \sum_{j} V_j^{D, buy}(t_0) - 1.3 \sum_{k} V_k^{D, sell}(t_0). \quad (8)
\]

The procedure of analytical approach for calculating TBP is summarized by the following algorithm:

**Algorithm 1. (Analytical Approach for TBP)**

**STEP 1**: Evaluate the market values of all the securities $V_i, i = 1, 2, \ldots, N$ (including the shorted ones) of the customer credit; calculate the covariance matrix
\[
\Sigma = [\text{cov}(\Delta V_i, \Delta V_j)]_{N \times N}
\]
according to daily draft \( (\mu_i) \) and volatility \( (\sigma_i) \) based on the historical data of the past 1 year.

**STEP 2:** Determine the value of \( K \) and \( P(t_0) \) via (3) and (8) respectively.

**STEP 3:** Calculate the mean value and standard deviation of daily increment \( \Delta P \) of portfolio \( P \):

\[
\mu(\Delta P, t_0)_{day} = \sum_{i=1}^{n} w_i \mu_i V_i(t_0), \quad \sigma(\Delta P)_{day} = \sqrt{W^T \Sigma W},
\]

where \( W = (w_1, w_2, \ldots, w_N)^T \) is the quantity coefficient vector.

**STEP 4:** Return the value of TBP via (7):

\[
TBP \leftarrow \Phi \left( \frac{K - P(t_0) - \Delta T \cdot \mu(\Delta P, t_0)_{day}}{\sqrt{\Delta T} \cdot \sigma(\Delta P, t_0)_{day}} \right).
\]

Here duration \( \Delta T \) has unit day.

*Remark 1:* the proposed analytical calculation method can output a sufficiently precise TBP within a small inter-arrival time \( \Delta t \). However, we can only get an approximate TBP with this approach in duration \( \Delta T \). This is exactly the same situation as the normal parameter method being employed to calculate VaR. In fact, the increments \( \Delta V_i \) under Geometric Brownian Motion are mutually independent, but not necessarily identically distributed, it depends on \( V_i \), e.g., the stock price change of each day depends on its opening price each day. Thus, although the assumption A4 does make the calculation of TBP simpler, it can only output sort of approximate results.

*Remark 2:* In the above Algorithm 1, it is also the assumption A4 that enable use to replace the original standard deviation with time \( t_0 \) with \( \sigma(\Delta P)_{day} \), i.e.,

\[
\sigma(\Delta P)_{day} = \sqrt{W^T \Sigma W} = \sigma(\Delta P, t_0)_{day} = \sqrt{\Delta t \sum_{i} \sum_{j} w_i w_j \rho_{ij} \sigma_i \sigma_j V_i(t_0) V_j(t_0)}.
\]

*Remark 3:* An observation of (7) is that the TBP is fully determined by the market value of the asset-liabilities of the customer credit account and distribution of its (long-short) portfolio. On the one hand, it follows from

\[
K - P(t_0) = 1.3(C_{D, buy}(0) + P_m) - C_A(t_0) - C_{D, sell}(0) - P(t_0)
\]

\[
= 1.3P_m + \left( 1.3C_{D, buy}(0) - \sum_j V_{j}^{D, buy}(t_0) \right) + \left( 1.3 \sum_k V_k^{D, sell}(t_0) - C_{D, sell}(0) \right) - C_A(t) - \sum_i V_i^A(t_0)
\]

that the larger loss of the account, the easier it will break the threshold, i.e., TBP will be higher (resp., the higher the expected return of the portfolio, the lower the TBP will be). On the other hand, when \( TBP > 0.5 \) (i.e., \( K - P(t_0) - \Delta T \cdot \mu(\Delta P, t_0)_{day} > 0 \)), the larger the portfolio volatility is, the lower probability that it will break the threshold (TBP becomes lower and closer to 0.5 from above, the smaller the risk); oppositely, when \( TBP < 0.5 \) (i.e., \( K - P(t_0) - \Delta T \cdot \mu(\Delta P, t_0)_{day} < 0 \)), the larger portfolio volatility will increase the risk, since in this case, the TBP will be higher and closer to 0.5 from blow. Interestingly, the volatility of the portfolio dominates the risk change (TBP variation) to different directions in different situations.
2.2. Empirical Frequency Approach for TBP. The previous analytical method, under the situation when historical default data are unavailable, can be employed to estimate the TBP on the basis of the distribution of portfolio increment of the customer account. The TBP obtained in such a way is essentially a “subjective probability” without investigating the sufficient historical default data. Hence, this subsection is intended to introduce an empirical frequency mapping method that fits in the situation when the default data set is accessible. The basic principle of this method is as simple as it could be: to map the “Threshold distance (TD)” into TBP.

First, we define the TD at time \( t_0 \) as the difference between the margin ratio at \( t_0 \) and the threshold 1.3:

\[
TD(t_0) = R(t_0) - 1.3
\]

where

\[
R(t) = \frac{C_A(t_0) + \sum_i V_i^A(t_0) + C_{D, sell}(0) + \sum_j V_j^{D, buy}(t_0)}{C_{D, buy}(0) + \sum_k V_k^{D, sell}(t_0) + P_m}.
\]

Then, TBP can be obtained via following formula:

\[
\text{TBP} = \frac{\text{The number of account samples that have } TD = TD(t_0) \text{ and then default in } \Delta T}{\text{The total number of account samples with } TD = TD(t_0)}.
\]

3. Default Probability. The TBP actually provides the probability of margin call. After the margin call being occurred, the customer’s choice to cover short position (do not default), or to give up the account and let it be forcibly liquidated (default) in general depends on the factors in the following two phases:

a. The loss level of the customer’s credit account. The more the account loses, the higher possibility that the customer will choose to “default”. I can be clearly seen from (7) that the TBP substantially measures the possibility that the potential loss level of the credit account exceeds certain “threshold”, the larger the potential loss level, the higher the TBP. Hence, the TBP can be utilized to reflect the potential loss level.

b. The credit rank of the individual customer. It is reasonable to believe that a customer with higher credit rank has a lower possibility to default compared with the customer with lower credit rank. Currently, most brokers with SMTSL (securities companies with SMTSL business) already established rating system for customers’ credit qualities. Here, we just pick a simple sample system that partitions the customers into 5 ranks (Table 1) by considering holistically different attributes of the customers (e.g., natural attribute, trading attribute, inner attribute and outer attribute).

<table>
<thead>
<tr>
<th>Credit Rank</th>
<th>Comprehensive Credit Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>5A</td>
<td>90≤S</td>
</tr>
<tr>
<td>4A</td>
<td>85≤S&lt;90</td>
</tr>
<tr>
<td>3A</td>
<td>80≤S&lt;85</td>
</tr>
<tr>
<td>2A</td>
<td>75≤S&lt;80</td>
</tr>
<tr>
<td>1A</td>
<td>70≤S&lt;75</td>
</tr>
</tbody>
</table>
Taking into the above two determinants on credit default, we define the default probability (DP) as follows:

\[ \text{DP} = \text{TBP} \cdot \min\{P(\text{Default} \mid \text{TB}) \cdot (\lambda K_{\text{loss}} + (1 - \lambda) K_{\text{credit}}), 1]\],

where

\[ K_{\text{loss}} = 0.5 + \text{TBP}, \]

\[ K_{\text{credit}} = 1 - \frac{S - 85}{30}. \]

\( P(\text{Default} \mid \text{TB}) \) is the empirical (conditional) frequency of default (this value could be a sort of average value) under the condition of threshold being violated; 

\( K_{\text{loss}} \) is the contribution of account loss to the default which builds on the TBP, and 

\( \lambda \) \((0 < \lambda < 1)\) is the contribution weight; 

\( K_{\text{credit}} \) is the contribution of customer credit quality to the default with \((1 - \lambda)\) being its weight.

Remark 5: A close observation of (9) could tell that this formula incorporates the individual default characteristics into the DP by adjusting the empirical conditional frequency (average default characteristics) with “potential loss extent” determinant and “customer credit quality” determinant.

Remark 6: Furthermore, from (10)-(11), when \(\lambda=1\) (the potential loss fully dominates the default possibility): if TBP=0.5, which means the potential loss lies in the “average level”, hence, the empirical conditional probability equals to the empirical conditional default frequency \(P(\text{Default} \mid \text{TB})\) (average level); if TBP>0.5, which means the potential loss exceeds the “average level”, therefore the empirical conditional probability should accordingly also higher than the “average level”, i.e., \(P(\text{Default} \mid \text{TB})\), with a given upper bound of 1.5 multiples of \(P(\text{Default} \mid \text{TB})\) and capped by 1 at the same time; the case of TBP<0.5 can be explained by the same logic, with a given lower bound of 0.5 multiples of \(P(\text{Default} \mid \text{TB})\). Similarly, when \(\lambda=0\), the contribution of customer credit quality to default also be within (0.5, 1.5), and when the customer credit quality is of “middle level” \((S=85)\), the conditional default probability equals to the empirical average value \(P(\text{Default} \mid \text{TB})\).

3. Loss given Default (LGD). Generally speaking, most current popular methods for evaluating the Loss given Default (LGD) on counterpart follow two steps as below (see Crouhy et. al., 2000): first, determine the Exposure at Default (EAD) to the counterpart; after that, determine the loss rate at default (LRD) based on the analysis of historical default data, and the expected loss given default is EAD*LRD. However, there are two stubborn difficulties to the calculations of EAD and LRD for the SMTSL business:

D1. The dynamics of EAD: the EAD of the broker to the customers is the “Assets-Liabilities” of the customer’s account, i.e.,

\[ \text{EAD}(T^*) = \max\{D(T^*) - A(T^*), 0\} \]

where \(T^*\) is the time spot of default, and

\[ D(T^*) = C_{D,\text{buy}}(0) + \sum_k V_{k}^{D,\text{sell}}(T^*) + P_{m}, \]

\[ A(T^*) = C_{A}(t_0) + \sum_i V_{i}^{A}(T^*) + C_{D,\text{sell}}(0) + \sum_j V_{j}^{D,\text{buy}}(T^*). \]

It therefore can be seen that the EAD at \(T^*\) is not static, but rather a distribution w.r.t. the market value of portfolio at \(T^*\). However, the most difficult part is, the default time spot \(T^*\) itself is also a random variable, it is related to the event of
“threshold violation”. Hence, the EAD(T*) consists a mathematical structure of two-fold probability distribution, which is not easy to tackle.

D2. Mixture of EAD and LRD: In the business of SMTSL, the loss rate at default (LRD) is also of dynamic characteristics that directly relate to the EAD. Hence, it is almost impossible to treat them separately in the common way.

To surmount the above two difficulties, two distinct methods are proposed to compute the loss given default: VaR approach and empirical frequency approach.

3.1. VaR Approach. The basic thought for this approach is: we don’t treat EAD and LRD separately, but rather estimate the Loss given default (LGD) directly in a robust manner (worst case preference). First, we determine the VaR at the threshold violation, and then determine the VaR of the market value loss during the time from threshold being broken to the completion of forced liquidation. Summation of the two as the LGD (in a conservative perspective).

Denote the corresponding loss by the time \( t_0 + \Delta t \) as
\[
V_{\text{loss}}(t_0 + \Delta t) = D(t_0) - A(t_0) + P_{\text{loss}}(\Delta t)
\]  
(15)

where
\[
P_{\text{loss}}(\Delta t) = \sum_k V_k^{D,\text{sell}}(\Delta t) - \sum_i V_i^A(\Delta t) - \sum_j V_j^{D,\text{buy}}(\Delta t).
\]

Therefore, the VaR with broken threshold (VaR_BT) can be determined by
\[
\text{VaR}_{\text{BT}} = \text{VaR}_\beta(V_{\text{loss}}(t_0 + \Delta T^*)(\omega)| P(t_0 + \Delta T^*)(\omega) \leq K),
\]  
(16)

where \( \omega \) is the random event, and \( t_0 + \Delta T^* \) is the default time spot.

From (15), \( \text{VaR}_{\text{BT}} \) can be estimated by following Ito Process with Monte Carlo Simulation:
\[
\Delta P = \mu(\Delta P, t) \Delta t + \sigma(\Delta P, t) \phi_i \sqrt{\Delta t}, \quad \phi_i \sim N(0,1),
\]
where \( \mu(\Delta P, t), \sigma(\Delta P, t) \) are given by (5).

Considering in the business practice, it normally takes 2 trading days from the threshold broken being identified through the margin call and then to the forced liquidation, we therefore determine the LGD by adding two day’s VaR of \( \Delta P_{\text{loss}} \) to the \( \text{VaR}_{\text{BT}} \), i.e.,
\[
LGD = \max\{\text{VaR}_{\text{BT}} + \sqrt{2} \cdot \text{VaR}_{0.9}(\Delta P_{\text{loss}}), 0\},
\]
(17)

here under assumptions A3-A4, \( \Delta P_{\text{loss}} \) obeys the following distribution (the parameters are with similar meanings as the corresponding distributions in Subsection 2.1):
\[
\Delta P_{\text{loss}}(\text{day}) \sim N\left(\sum_{i=1}^n \mu_i w_i^i V_i(t_0), \sqrt{w^\mu \Sigma w'}\right).
\]
(18)

Remark 7: The independent and identical distribution assumption is also utilized here for \( \Delta P_{\text{loss}} \) to calculate the second VaR.

3.2. Empirical Frequency Approach. Analogously as in Subsection 2.2, we can design a empirical frequency approach to estimate the LGD when the loss date at default are available.

Since it is impossible to determine the EAD at the default time spot, we consider the asset-liability difference at \( t_0 \) as the EAD at \( t_0 \):
\[
\text{CE}(t_0) = \max\{D(t_0) - E(t_0), 0\},
\]
(19)
where

\[ E(t_0) = C_A(t_0) + \sum_i V_i^A(t_0). \]  (20)

Based on the historical data on loss at forced liquidation, we can get the distribution of

\[ q(\omega) = \frac{V_{\text{loss}}(T^{**})(\omega)}{CE(t_0)(\omega)}. \]  (21)

where \( \omega \) is random sample, \( T^{**} \) is the time when forced liquidation is complete. Furthermore, according to the distribution of \( q(\omega) \), we can calculate LGD with following formula:

\[ \text{LGD} = E[q(\omega)] * CE(t_0). \]  (22)

4. **Measuring the Default Correlation.** Making use of default probability (DP) and loss given default (LGD), we can calculate the default correlation by following formula:

\[ \rho_{ij} = \frac{\text{cov}(LGD_i, LGD_j)}{\sigma_{LGD_i} \cdot \sigma_{LGD_j}} = \frac{DP(i, j) - DP_i DP_j}{\sqrt{DP_i (1 - DP_i) DP_j (1 - DP_j)}}, \]  (23)

where \( DP(i, j) \) is the joint default probability of customers \( i, j \), which can be estimated using TBP\((i, j)\):

\[ \text{TBP}(i, j) = \Phi_2(\Phi^{-1}(TBP_i), \Phi^{-1}(TBP_j), \rho_{MV}(i, j)), \]  (24)

where \( \Phi_2 \) is accumulated distribution function of bivariate normal distribution, and \( \rho_{MV}(i, j) \) is the correlation coefficient between the market values of portfolios \( P_i \) and \( P_j \) of the credit accounts of customers \( i, j \).

5. **Calculating the Economic Capital.** The expected loss (EL) of single customer (account) \( i \) is the expected value of loss given default:

\[ EL_i = LGD_i \cdot DP_i, \]  (25)

and the unexpected loss (UL) as the standard deviation can be obtained by the following formula:

\[ UL_i = LGD_i \cdot \sqrt{DP_i (1 - DP_i)}. \]  (26)

As for the situation of multi-customers, the expected loss is

\[ EL_P = \sum_{i=1}^{n} EL_i, \]  (27)

and unexpected loss is

\[ UL_P = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} UL_i UL_j \rho_{ij}}, \]  (28)

where \( \rho_{ij} \) is the default correlation. For more detailed information on above formulas, the readers may refer to (Crosbie, A. Kocagil, 2003).

6. **A Simple Case Study.** We consider a customer credit account on the date of 2012-08-27 as in below two tables (Tables 2 and 3):
TABLE 2. Customer account information-I

<table>
<thead>
<tr>
<th>Horizon (Year)</th>
<th>Case (t0)</th>
<th>Securities</th>
<th>Code</th>
<th>Quantity</th>
<th>Market value (t0)</th>
<th>Price (t0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.45</td>
<td>3150.46</td>
<td>SA</td>
<td>600160</td>
<td>619000</td>
<td>5379110</td>
<td>8.69</td>
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<td></td>
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<td>32604</td>
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<td></td>
<td></td>
<td>SC</td>
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<td>470000</td>
<td>5029000</td>
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<td></td>
<td></td>
<td>SD</td>
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<td>10782</td>
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<td></td>
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<td>SE</td>
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<td>1307847</td>
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<tr>
<td></td>
<td></td>
<td>Total</td>
<td></td>
<td></td>
<td>30497978.04</td>
<td></td>
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</table>

TABLE 3. Customer account information-II

<table>
<thead>
<tr>
<th>Financing Amount (t0)</th>
<th>10323164</th>
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</thead>
<tbody>
<tr>
<td>Shorted Securities borrowed</td>
<td>Code</td>
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<tr>
<td>SF</td>
<td>600900</td>
</tr>
<tr>
<td>C_D(t0)</td>
<td>66100</td>
</tr>
<tr>
<td>Interests &amp; Premium</td>
<td>16500</td>
</tr>
<tr>
<td>K</td>
<td>13372312</td>
</tr>
</tbody>
</table>

We calculate the return rate (Table 4), return (Table 5), and the covariance matrix (of the security portfolio, Table 6) with the historical data in the past 1 year:

TABLE 4. Return rate

<table>
<thead>
<tr>
<th>Stock name</th>
<th>Code</th>
<th>Mean</th>
<th>Volatility</th>
<th>Price(t0)</th>
<th>Weight_P</th>
<th>Product_P</th>
<th>Weight_loss</th>
<th>Product_Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA</td>
<td>600160</td>
<td>-0.0040</td>
<td>0.0373</td>
<td>8.69</td>
<td>619000</td>
<td>-21690.99</td>
<td>-619000</td>
<td>21690.987</td>
</tr>
<tr>
<td>SB</td>
<td>600348</td>
<td>-0.0023</td>
<td>0.0258</td>
<td>14.3</td>
<td>2280</td>
<td>73.4775</td>
<td>-2280</td>
<td>73.47751</td>
</tr>
<tr>
<td>SC</td>
<td>600502</td>
<td>-0.0010</td>
<td>0.0255</td>
<td>10.7</td>
<td>470000</td>
<td>-5186.73</td>
<td>-470000</td>
<td>-5186.73</td>
</tr>
<tr>
<td>SD</td>
<td>601088</td>
<td>-0.0008</td>
<td>0.0159</td>
<td>21.36</td>
<td>10782</td>
<td>-178.39</td>
<td>-10782</td>
<td>-178.39</td>
</tr>
<tr>
<td>SE</td>
<td>562</td>
<td>0.0005</td>
<td>0.0273</td>
<td>21.36</td>
<td>1307847</td>
<td>-9816.16</td>
<td>-1307847</td>
<td>9816.16</td>
</tr>
</tbody>
</table>

TABLE 5. Return
TABLE 6. Covariance Matrix

<table>
<thead>
<tr>
<th>Cov Matrix</th>
<th>SA</th>
<th>SB</th>
<th>SC</th>
<th>SD</th>
<th>SE</th>
<th>SF</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA</td>
<td>0.5838</td>
<td>0.1527</td>
<td>0.10867</td>
<td>0.1119</td>
<td>0.1083</td>
<td>0.01322</td>
</tr>
<tr>
<td>SB</td>
<td>0.1527</td>
<td>0.2291</td>
<td>0.0905</td>
<td>0.1551</td>
<td>0.1041</td>
<td>0.01404</td>
</tr>
<tr>
<td>SC</td>
<td>0.1087</td>
<td>0.0905</td>
<td>0.1193</td>
<td>0.0816</td>
<td>0.0678</td>
<td>0.00970</td>
</tr>
<tr>
<td>SD</td>
<td>0.1119</td>
<td>0.1551</td>
<td>0.0816</td>
<td>0.1638</td>
<td>0.0871</td>
<td>0.01437</td>
</tr>
<tr>
<td>SE</td>
<td>0.1083</td>
<td>0.1041</td>
<td>0.0678</td>
<td>0.0871</td>
<td>0.1721</td>
<td>0.00954</td>
</tr>
<tr>
<td>SF</td>
<td>0.0133</td>
<td>0.0140</td>
<td>0.0097</td>
<td>0.0144</td>
<td>0.00954</td>
<td>0.00381</td>
</tr>
</tbody>
</table>

Coding the proposed models in Matlab, we can easily get all the results on the credit risk of this customer account:

TABLE 7. Credit risk result

<table>
<thead>
<tr>
<th>Var_delta_P(day)</th>
<th>8.71745E+11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std Dev_delta_P(day)</td>
<td>933672.6374</td>
</tr>
<tr>
<td>Mean_delta_P(day)</td>
<td>-17327.54413</td>
</tr>
<tr>
<td>P(t_0)</td>
<td>30413348.04</td>
</tr>
<tr>
<td>K</td>
<td>13372312.44</td>
</tr>
<tr>
<td>delta T(day)</td>
<td>126</td>
</tr>
<tr>
<td>Normal_Para</td>
<td>-1.417664885</td>
</tr>
<tr>
<td>Analytical TBP</td>
<td>7.81%</td>
</tr>
<tr>
<td>Simulation based TBP</td>
<td>3.50%</td>
</tr>
<tr>
<td>K_loss</td>
<td>0.535</td>
</tr>
<tr>
<td>K_credit</td>
<td>0.833333333</td>
</tr>
<tr>
<td>S</td>
<td>90</td>
</tr>
<tr>
<td>Conditonal DP</td>
<td>0.2</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.5</td>
</tr>
<tr>
<td>DP</td>
<td>0.004789167</td>
</tr>
<tr>
<td>D(t_0)</td>
<td>10404763.77</td>
</tr>
<tr>
<td>A(t_0)</td>
<td>30567228.5</td>
</tr>
<tr>
<td>V_loss(t_0)</td>
<td>-20162464.73</td>
</tr>
<tr>
<td>VaR given default</td>
<td>-2235400</td>
</tr>
<tr>
<td>delta_VaR(2)_0.99</td>
<td>-3106393.916</td>
</tr>
<tr>
<td>LGD</td>
<td>870993.916</td>
</tr>
<tr>
<td>EL</td>
<td>417133503</td>
</tr>
<tr>
<td>UL</td>
<td>60131.58402</td>
</tr>
</tbody>
</table>

REFERENCE

