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HOW TO INCENTIVIZE STUDENTS TO GRADUATE FASTER

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ABSTRACT. *At prime research universities, students study full-time and receive their Bachelors's degree in four years. In contrast, at urban universities, many students study only part-time, and take a longer time to graduate. The sooner such a student graduates, the sooner will the society start benefiting from his or her newly acquired skills – and the sooner the student will start earning more money. It is therefore desirable to incentivize students to graduate faster. In the present paper, we propose a first-approximation solution to the problem of how to distribute a given amount of resources so as to maximally speed up students graduation.*

Keywords: Speeding Up Graduation; The General Decision Making Theory; Incentive

1. Formulation of the Practical Problem.

1.1. Fact: Students at Urban Universities often Take Longer to Graduate. At prime research universities, students study full-time and receive their Bachelor's degrees in four years. In contrast, at urban universities, many students study only part-time. As a result, these students take longer to graduate.

1.2. Speeding up Graduation Is a Win-win Idea. From the viewpoint of the student, the sooner he or she graduates, the sooner will his or her salary increase reflecting the newly acquired skills.

From the viewpoint of the society as a whole, the sooner a student graduates, the sooner will the society start benefiting from his or her newly acquired skills. In other words, speeding up graduation is a win-win idea.

1.3. How Can We Speed up Graduation. Among the main reasons why some students at urban universities only study part-time are financial reasons. So, to speed up student graduation, it is desirable to provide financial incentives.

1.4. Towards a Corresponding Optimization Problem. In the ideal world, we should be able to fully support every student. In real life, however, our resources are limited. So, the question is: what is the best way to distribute these resources so that we can maximally speed up student graduation – or, equivalently, maximally increase the number of classes n that a student takes every semester.This is a problem that we will be solving in this paper.

2. Let Us Reformulate the above Practical Problem in Precise Terms.

2.1. Modeling Student's Decisions. In order to solve the above problem, let us first formulate it in precise terms. According to the general decision making theory (Fishburn, 1969; Luce and Raiffasee, 1989; Raiffa, 1997), every agent selects a decision that maximizes his or her utility *u*. So, to understand the student's behavior, we need to understand how this utility *u* depends on the number *n* of classes per semester.

In general, it is reasonable to assume that this dependence $u(n)$ is smooth –even analytical, so the dependence can be well approximated by a Taylor series $u(n) = u_0 + u_1 \cdot n + u_2 \cdot n^2 + \ldots$. The number of hours *n* does not differ that much between different students, so the range of *n* is small, and on a small range, a few first terms in the Taylor expansion are usually sufficient to reasonably accurately describe the dependence. Let us see how many terms we need for our problem.

The 0-th order term u_0 can be interpreted as a utility of simply being at a university. Since this term does not depend on the number of classes *n* that a student is taking, it does not affect the student's decision. Therefore, we can safely ignore this term and assume that $u_0 = 0$.

The next term $u_0 \cdot n$ represents the gain in knowledge (minus effort) per class. While there are minor differences in how much material students learn, in the first approximation, it is reasonable to assume that this amount is approximately the same for all the students. Since maximizing the function $u(n) = u_1 \cdot n + \ldots$ and maximizing the function $\frac{u(n)}{u_1} = n + \ldots$

are equivalent tasks, we can safely assume that $u_1 = 1$, i.e., that the linear term has the form $u(n) = n$.

If we only had this linear term, then the more classes the student would take, the larger this student's utility. In other words, in this approximation, a student would take as many classes as there are available. This is clearly not what we observe. This means that in order to explain the actual student behavior, it is not sufficient to only consider linear terms in the dependence $u(n)$, we need to consider at least the terms of the next order – i.e., quadratic terms. Thus, we arrive at the utility expression $u(n) = n + u_1 \cdot n^2$.

If $u_1 > 0$, then this expression increases with *n* and so, we face the same problem as before. So, to explain the actual student behavior, we need to assume that $u_1 < 0$. In this case, the utility function has a clear maximum: when $\frac{du}{dn} = 1 + 2u_1 \cdot n = 0, i.e.$ when $n = \frac{1}{2|u_1|}$, for each student, we observe the actual number of classes n_a that this student takes, so we can conclude that for this student, $u_1 = -\frac{1}{2n_a}$ and thus, the student's utility function has the form:

$$u(n) = n - \frac{1}{2n_a} \cdot n^2$$

2.2. Modeling Student Population. In the above first approximation model, decisions by each student are characterized by a single parameter *na* – the number of classes that this student takes. Thus, to describe a student population, it is sufficient to describe the distribution of this parameter. This distribution can be described, e.g., by the probability density $\rho(n)$ which is defined, as usual, as the ratio $\frac{P(n \le n_a \le n+\Delta n)}{\Delta n}$ of the proportion $P(n \le n_a \le n+\Delta n)$ of students for whom the actual number of classes *na* is between *n* and *n* + Δ and the width Δ*n* of the corresponding interval [*n*, *n* + Δ*n*].

2.3. Adding an Incentive. A natural incentive is to give a discount for each course above a certain threshold n_0. This incentive adds, to the original utility, a new term $k \cdot (n - n_0)$, where $k > 0$ is the per-course value of this discount.

Once we select a threshold *n*0, we can determine the per-course discount value *b* by equating the total discount the total amount of offered discounts to the available amount *A*, i.e., from the condition that

$$k \cdot \int_{n_0} \rho(x) \cdot (n - n_0) dn = A.$$

From this condition, we can describe the value *k* as follows:

$$k = \frac{A}{\int_{n_0} \rho(x) \cdot (n - n_0) dn}$$

2.4. Decision Making in the Presence of this Incentive. Once we add the incentive, for $n < n_0$, we get the same utility as before, but for $n > n_0$, we get a new utility expression

$$u_i(n) = n - \frac{1}{2n_a} \cdot n^2 + k \cdot (n - n_0)$$

As a result, a student who previously selected n_a courses will now optimize a new objective function $u_i(a)$ and get a new number of courses n_i.

2.5. Our Objective. We want to select a threshold n_0 in such a way that the average increase in the number of courses is the largest possible, i.e., that the value is the largest possible.

$$\int_{n_0} \rho(n_a) \cdot (n_i(n_a) - n_a) dn_a$$

Now, the problem has been reformulated in precise terms, so we can start solving it.

3. Towards a Solution to the Problem. Differentiating the new utility function $u_i(n)$ with respect to *n* and equating the derivative to 0, we conclude that for the value n_i at which this objective function attains its maximum, we get

$$1 - \frac{n_i}{n_a} + k = 0,$$

hence $n_i = (1+k)\cdot n_a$.

Thus, $n_i(n_a) - n_a = k\cdot n_a$ and so, the objective function that we use to select a threshold n_0 takes the form:

$$k\int_{n_a} \rho(n)\cdot n dn$$

Substituting the above expression for *k*, we conclude that we need to maximize the following expression:

$$A\cdot\frac{\int_{n_a}\rho(n)\cdot n dn}{\int_{n_a}\rho(n)\cdot(n-n_0)dn}$$

Dividing the objective function by a constant does not change the value at which this function attains its maximum. So, the above maximization problem is equivalent to the problem of maximizing the following ratio:

$$\frac{\int_{n_a}\rho(n)\cdot n dn}{\int_{n_a}\rho(n)\cdot(n-n_0)dn}$$

Maximizing an expression E is equivalent to minimizing its reciprocal $\frac{1}{E}$. Thus, maximizing the above ratio is equivalent to minimizing the reciprocal ratio

$$\frac{\int_{n_a}\rho(n)\cdot(n-n_0)dn}{\int_{n_a}\rho(n)\cdot n dn}$$

Here,

$$\int_{n_a}\rho(n)\cdot(n-n_0)dn = \int_{n_a}\rho(n)\cdot n dn - n_0\cdot\int_{n_a}\rho(n)dn$$

and therefore, the above ratio takes the form

$$1-\frac{n_0\cdot\int_{n_a}\rho(n)dn}{\int_{n_a}\rho(n)\cdot n dn}$$

Minimizing this expression $1-r$ is equivalent to maximizing r, i.e., to minimizing the ratio:

$$\frac{n_0\cdot\int_{n_a}\rho(n)dn}{\int_{n_a}\rho(n)\cdot n dn}$$

This minimization, in its turn, is equivalent to maximizing the reciprocal ratio:

$$\frac{1}{n_0}\frac{\int_{n_a}\rho(n)\cdot n dn}{\int_{n_a}\rho(n)dn}$$

One can easily check that the ratio:

$$\frac{\int_{n_a}\rho(n)\cdot n dn}{\int_{n_a}\rho(n)dn}$$

is, by definition, equal to the conditional mean of the variable *n* under the condition that

$n \ge n_0$:

$$\frac{\int_{n_a} \rho(n) \cdot n dn}{\int_{n_a} \rho(n) dn} = E[n | n \ge n_0]$$

Thus, we arrive to the following conclusion.

4. Solution to the Optimization Problem.

4.1. Solution. To maximize the effect of the incentive, we should select a threshold n_0 for which the following ratio is the largest possible:

$$\frac{E[n | n \ge n_0]}{n_0}$$

4.2. Discussion. For distributions with "light" tails – similar to the normal distribution – the above ratio decreases with n_0. Thus, for such distributions, to achieve the largest effect, we should select the smallest possible threshold n_0.

For heavy-tailed distributions (Mandelbrot, 1983; Resnick, 2007), for the Pareto distribution, when $\rho(n) = C \cdot n^{-\alpha}$ for all $n \ge N$ for some small N – the situation is different. For example, for the Pareto distribution, the above ratio does not depend on the threshold n_0; therefore, to decide which threshold to select, it is not sufficient to use the above first approximation: we must consider the next approximation as well.

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