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FAULT DIAGNOSIS BASED ON THE NARX MODEL IN THE FREQUENCY DOMAIN FOR ROTARY MACHINE

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ABSTRACT. *This paper deals with a fault diagnosis method based on a nonlinear spectral analysis known as generalized frequency response functions (GFRFs) for a bearing and shaft machine. The nonlinear autoregressive exogenous (NARX) model identifies proper model using genetic algorithm for input-output data in normal and two faulty working states of the rotary machine. The GFRFs are applied to convert polynomial NARX models from time domain to frequency domain. After feature extraction, all working states are classified to individual batches using principle component analysis. The results of third order GFRFs clearly show that the method is able to classify faults as well as healthy conditions of the rotary machine in frequency domain.*

Keywords: Fault Diagnosis; GFRF; NARX Model; Frequency Domain; Rotary Machine

1. Introduction. Signal analysis (model free) and model based methods are two main approaches to diagnose the faults in a system. In model free methods the signal model is used for fault diagnosis rather than mathematical model of system. However these methods only use output signals of system, therefore the effect of input on the output is not considered. An explicit mathematical model of the plant uses in model based fault diagnosis techniques. There is an increasing interest in theory and application of model based fault detection and diagnosis methods. The main reasons are safety, economical and ability to perform on-line as well as these methods use both input and output signal of the system.

One method to diagnose the fault in a system is based on the nonlinear spectral analysis, which has developed in recent years. The concept of generalized frequency response functions (GFRFs) is the basis nonlinear systems in the frequency domain (George 1959). GFRFs are recognized as one of the most powerful and successful tools to describe system behaviors by both analytical and graphical means. In comparison with the traditional fault diagnosis method based on signal analysis for the mechanical faults, this method is more efficient, in which the fault would cause the obvious nonlinear change of the transfer characteristic of the system.

In this issue, the concept of linear frequency response function expresses the nonlinear cases which they have fading memory and are described by the Volterra series model (Boyd et al. 1983). After deriving the nonlinear model from given data, the GFRFs are calculated. Comparison among GFRFs for different modes helps to diagnose and recognize the faults in a system. However, the application of this technique is limited, because it is

not easy to identify GFRFs in the frequency domain. Wei et al. (2002) proposed method based on the multiple preset GFRF model and applied it on the vehicle appending system. However, the method is not flexible enough for all applications. Tang et al. (2010) applied Volterra modeling to model three different states of rotary machine using GFRFs in the frequency domain. The key problem of nonlinear system modeling in frequency domain is the converting model from time to the frequency domain. In Volterra modeling, regressor coefficients, kernels, have a direct physical significance. The nth order transfer function of GFRF can be expressed by the concept of multidimensional Fourier transformation of the nth order impulse response function. Thus, their function is restricted to use Volterra model. For other polynomial models the procedure of deriving GFRFs is not straightforward as Volterra model. However, polynomial models have their own advantages to use as a nonlinear model structure. The advantage of NARX models consists in availability of various methods for their approximation and simplicity of control design as well as its simplicity and fast convergence of identification. This type of model has broad applications such as prediction, simulation and identification. However, the performance of this model is highly dependent on the model structure selection, which in this study, we select them properly using the genetic algorithm.

In the present study, a model based fault diagnosis approach using polynomial NARX model in the frequency domain for different faulty states of a rotary machine is presented. The identification of the NARX model structure is carried out using genetic algorithm. The faults of induction motor are broken rotor bar and stator winding faults that will be detected and classified using principal components analysis (PCA) of GFRFs features.

2. NARX Model. Throughout this study, polynomial NARX model is used to model the given data sets of various working state of bearing and shaft. This type of model has broad applications such as prediction, simulation and identification. The main issue in identification of these polynomial models is finding the order of the polynomial and choosing the significant terms from a large collection of combination sets that exist for this model representation. A wide class of nonlinear systems with integrated representation can be presented by NARX model. The model is stated as

$$y(t) = f^{n_l}\left(u(t-1), \dots, u(t-n_u), y(t-1), \dots, y\left(t-n_y\right)\right) + e(t) \tag{1}$$

where n_u and n_y are the corresponding maximum lags for input and output, $e(t)$ explain the uncertainties and possible noise, n_l is the degree of non- linearity and f^{n_l} can be assumed as a variety of nonlinear function forms based on n_l. The identification of the nonlinear system is more difficult compared with linear model since it involves more terms. The expanded form of NARX model is the summation of terms with the degree of nonlinearity ($1 \le n_l \le L$) as follows:

$$y(t) = \sum_{m=1}^{M} y_m(t)$$

$$y_m(t) = \sum_{p=0}^{m} \sum_{k_1, k_{p+q}=1}^{k} c_{p,q}\left(k_1, \dots, k_{p+q}\right) \times \prod_{i=1}^{p} y(t-k_i) \prod_{i=p+1}^{p+q} u(t-k_i) \tag{2}$$

where, $y_m(t)$ is the mth order output of the system, and

$$p+q=m, \;\; k_i = 1, \dots, K, \;\; \sum_{k_1, k_{p+q}=1}^{K}(.) = \sum_{k_1=1}^{K}(.) \dots \sum_{k_{p+q}=1}^{K}(.)$$

The parameters $c_{0,1}(.)$ and $c_{1,0}(.)$ represent the linear system parameters corresponding to the linear terms in the model, and all other represent the nonlinear system parameters. A nonlinear parameter $c_{p,q}(.)$ corresponds to a nonlinear term in the model of the pq^{th} degree of the form

$$\prod_{i=1}^{p} y(t-k_i) \prod_{i=p+1}^{p+q} u(t-k_i) \quad e.g.\ y(t-1)^p u(t-2)^q$$

2.1. NARX Model in the Frequency Domain. The concept of generalized frequency response functions was proposed by George (1959). The GFRFs extends the linear frequency response function to the non-linear case. Peyton-Jones and Billings (1989, 1990), presented a recursive algorithm for calculating the GFRFs of the NARX model, as well as they extended the result to nonlinear systems described by nonlinear integro-differential equations. Lang and Billings (1996, 1997) proposed the output frequency characteristics of nonlinear systems. The bound characteristics of the GFRFs and the output spectrum of the NARX model relationship between the model parameters and the magnitude of the system frequency response functions investigated in Zhang and Billings (1996) and Billings and Lang (1996). In the present study, we use the result of the work by Peyton-Jones and Billings (1989) and Jing et al. (2006).

Consider the nonlinear systems, which can be described by NARX as shown in (2). The following recursive algorithm was derived in Peyton-Jones and Billings (1989) and Jing et al. (2006) to compute the GFRFs:

$$\begin{aligned} L(n).H_n(j\omega_1,\dots,j\omega_n) &= \sum_{k_1,k_n=1}^{k} c_{0,n}(k_1,\dots,k_n)\exp(-j(\omega_1 k_1+\cdots+\omega_n k_n)) \\ &+\sum_{q=1}^{n-1}\sum_{p=1}^{n-q}\sum_{k_{p+q}=1}^{K} c_{p,q}(k_1,\dots,k_{p+q}) \times \exp(-j(\omega_{n-q+1}\tau_{n-q+1}+\cdots+\omega_{p+q}\tau_{p+q})) \\ &\times H_{n-q,p}(j\omega_1,\dots,j\omega_{n-q}) \\ &+\sum_{p=2}^{n}\sum_{k_1,k_p=1}^{K} c_{p,0}(k_1,\dots,k_p)H_{n,p}(j\omega_1,\dots,j\omega_n) \end{aligned} \tag{3}$$

where

$$H_{n,p}(.)=\sum_{i=1}^{n-p+1} H_i(j\omega_1,\dots,j\omega_i)H_{n-i,p-1}\times(j\omega_{i+1},\dots,j\omega_n)\times\exp\left(-j(\omega_1+\cdots+\omega_n)k_p\right) \tag{4}$$

$$H_{n,1}(j\omega_1,\dots,j\omega_n)=H_n(j\omega_1,\dots,j\omega_n)\times\exp(-j(\omega_1+\cdots+\omega_n)k_1) \tag{5}$$

and

$$L(n)=1-\sum_{k_1}^{K} c_{1,0}(k_1)\times\exp(-j(\omega_1+\cdots+\omega_n)k_1 \tag{6}$$

For instance, for first, second and third order we have,

$$L(1)H_1(j\omega_1)=\sum_{k_1,k_n=1}^{K} c_{0,1}(k_1)\exp(-j\omega_1 k_1) \tag{7}$$

$$\begin{aligned} L(2)H_2(j\omega_1,j\omega_2) &= \sum_{k_1,k_2=1}^{K} c_{0,2}(k_1,k_2)\exp\left(-j(\omega_1 k_1+\omega_2 k_2)\right) \\ &+\sum_{k_1,k_2=1}^{K} c_{1,1}(k_1,k_2)\exp\left(-j(\omega_2 k_2)\right)H_{1,1}(j\omega_1) \\ &+\sum_{k_1,k_2=1}^{K} c_{2,0}(k_1,k_2)\,H_{2,2}(j\omega_1,j\omega_2)=\sum_{k_1,k_2=1}^{K} c_{0,2}(k_1,k_2)\exp\left(-j(\omega_1 k_1+\omega_2 k_2)\right)\times H_1(j\omega_1)\exp\left(-j(\omega_1 k_1)\right) \\ &+\sum_{k_1,k_2=1}^{K} c_{2,0}(k_1,k_2)\,H_1(j\omega_2)\,H_1(j\omega_1)\times\exp\left(-j(\omega_1 k_1+\omega_2 k_2)\right) \end{aligned} \tag{8}$$

$$L(3)H_3(j\omega_1, j\omega_2, j\omega_3) = \sum_{k_1,k_3=1}^{K} c_{0,3}(k_1, k_2, k_3) \times \exp\left(-j(\omega_1 k_1 + \omega_2 k_2 + \omega_3 k_3)\right)$$
$$+ \sum_{q=1}^{2} \sum_{p=1}^{3-q} \sum_{k_1,k_3=1}^{K} c_{p,q}(k_1 + \cdots + k_{p+q}) \times \exp(-j(\omega_{3-q+1} k_{3-q+1} + \cdots$$
$$+ \omega_{p+q} k_{p+q})) H_{3-q,p}(j\omega_1, \ldots, j\omega_{3-q}) \sum_{p=2}^{3} \sum_{k_1,k_p=1}^{K} c_{p,0}(k_1, \ldots, k_p) H_{3,p}(j\omega_1, j\omega_2, j\omega_3) \quad (9)$$

First and last terms in (3) show the pure inputs and outputs nonlinearities with $c_{0,n}(.)$ and $c_{n,0}(.)$ as well as the second one indicates input–output cross nonlinearities corresponding to the nonlinear parameters $c_{p,q}(.)$.

3. Diagnosis Approach. There are three major parts of the diagnosis approach. The first step is identification of the model using polynomial NARX model. The data sets are acquired by monitoring the motor current. The NARX model structure is identified using the genetic algorithm (Loghmanian et al., 2011), which explanation is not purpose of the current paper. Second step is converting NARX model in each state from the time domain to the frequency domain using equation (3). Feature extraction and pattern classification with principal components analysis (PCA) classify all faults. Figure 1 shows the procedure of the fault diagnosis using NARX model.

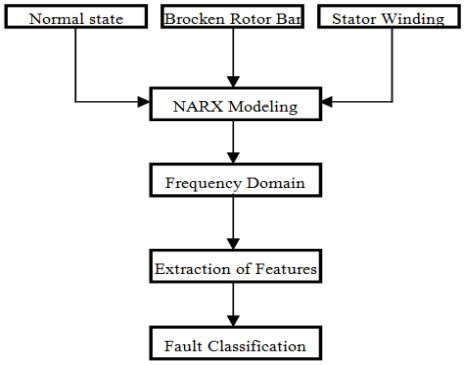

FIGURE 1. Flowchart of methodology

4. Experiment and Result. To test the performance and capability of the proposed fault diagnostic method and show how the method is being applied on a rotary machine the experiment is carried out using a rotor-bearing system and monitoring the motor current. Broken rotor bar and stator winding faults are two different working states, which have been investigated. Initially, the motor starts on normal condition. Thus, three different data sets are captured and simulated using NARX models as shown in Figure 2 as well as their NARX models are shown in equations (10) until (12) for healthy state, broken rotor and winding stator faults respectively.

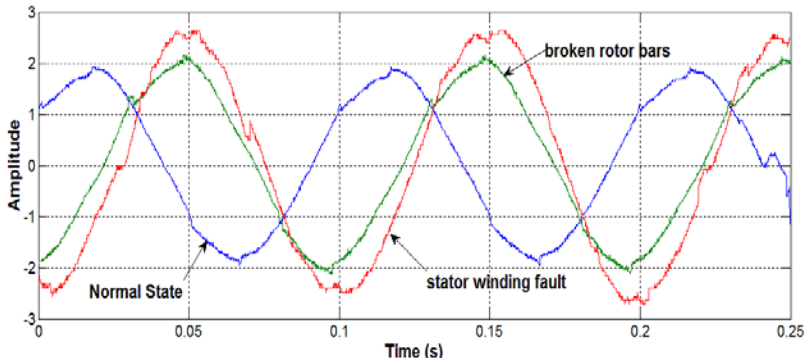

FIGURE 2. Input-Output of three different states of rotor-bearing system

$$y(t) = 1.21y(t-1) + 0.23y(t-2) + 0.01u(u-1) - 0.69y(t-1)y(t-2) + 0.68y(t-1)y(t-3) + 0.33y(t-2)^2 - 0.44y(t-3)u(t-1) + 0.44y(t-3)u(t-2) \quad (10)$$

$$y(t) = 2.52y(t-2) - 1.05y(t-4) - 0.01u(u-2) + 0.91y(t-1)u(t-1) - 0.91y(t-1)u(t-2) - 1.17y(t-2)u(t-1) + 0.78y(t-2)u(t-2) + 0.38y(t-2)u(t-2) + 0.61y(t-4)u(t-3) - 0.61y(t-4)u(t-4) \quad (11)$$

$$y(t) = 1.37y(t-2) - 0.37y(t-3) + 0.02u(u-2) - 0.34y(t-2)y(t-3) + 0.31y(t-2)u(t-4) + 0.26y(t-3)u(t-3) - 0.25y(t-3)u(t-4) + 0.67y(t-2)u(t-1) + 0.32y(t-4)u(t-4) \quad (12)$$

The generalized frequency response functions of all models are calculated using equation (3) to (9) up to third order. The selection of features is the important step to classify the faults. There are various features such as the average, maximum, root mean square, mean square and variance of amplitude, which are extracted from each order of GFRF model as the inputs for the PCA. The result of PCA for the first, second and third orders of GFRFs, H_1, H_2 and H_3 are shown in Fig. 3 until 5. The graph clearly shows that the normal state and faulty states are not recognizable to each other. However, combination of all three orders of GFRFs, H_1, H_2 and H_3, indicated the distinct classification as shown in Fig. 6. The graph shows that for rotor-bearing system, GFRFs of first three orders are sufficient to diagnose the faults.

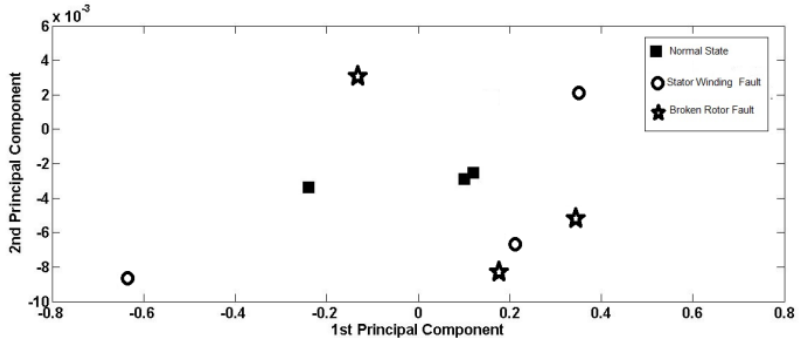

FIGURE 3. PCA for the first order of GFRFs for three cases

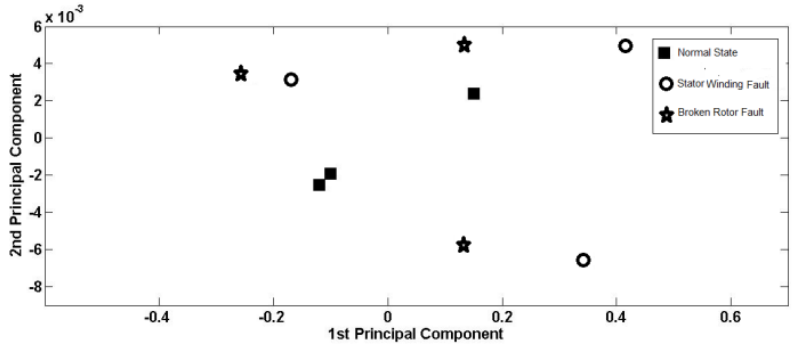

FIGURE 4. PCA for the second order of GFRFs for three cases

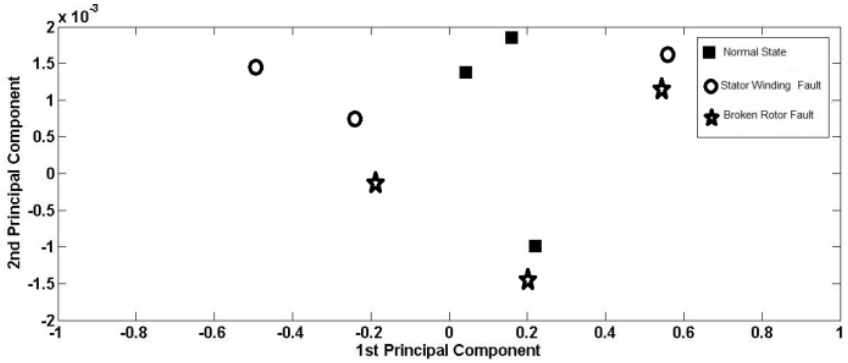

FIGURE 5. PCA for the third order of GFRFs for three cases

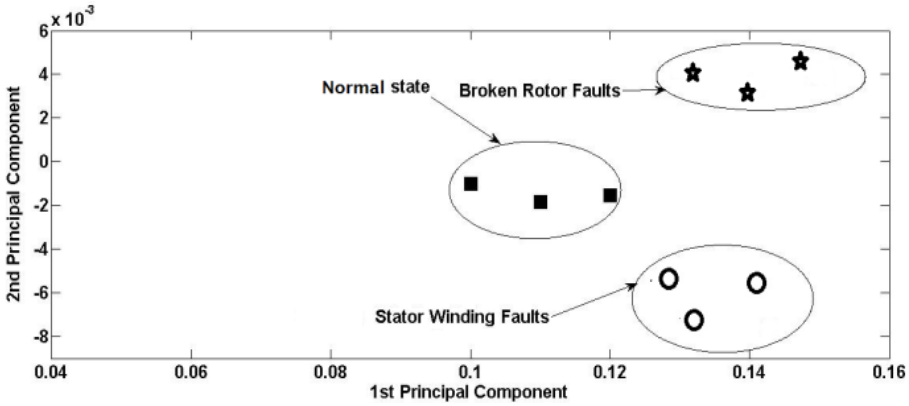

FIGURE 6. PCA for the combination of all three orders of GFRFs for three cases

5. Conclusions. A fault diagnosis method based on the nonlinear spectral analysis, generalized frequency response functions, was proposed using polynomial NARX model in current paper. The proposed approach is a model based fault diagnosis. The case study was a rotary machine with two different faults namely, broken rotor and stator winding faults. These two faults along with normal working state were clearly classified using the combination of GFRFs orders up to third order, while each order alone could not recognize the states from each other.

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