THE APPLICATION OF GREY SYSTEM THEORY TO EXCHANGE RATE PREDICTION IN THE POST-CRISIS ERA

HONG WU¹ AND FUZHONG CHEN²
¹School of Economy and International Trade
Zhejiang University of Finance and Economics
Hang Zhou, 310018, China
Wuhong71@163.com

²School of Business
Renmin University of China
Beijing, 100872, China
Chenhuzhong22@126.com

ABSTRACT. The international financial crisis has significantly affected every economy in the world. After the financial crisis, there are many factors together that aggravate the uncertainty of the Chinese exchange rate. A GM (1, 1) model is proposed for predicting short-term changes in the Chinese exchange rate. The Weakening Operator method is applied to improve the prediction accuracy. The results showed that the Grey System model has high accuracy in predicting the trends of change in the exchange rate with insufficient data. Based on the simulation results, the robustness of the model is supported. Additionally, the results also show that the Chinese exchange rate of RMB to USD is expected to appreciate.

Keywords: Grey System Theory; GM (1, 1); Exchange Rate Prediction; Post-Crisis Era

1. Introduction. Since 2008, financial crisis has been spread from USA to total world. The global financial crisis has brought great impact to every country in the world. The Federal Reserve Bank dropt its interest rate sharply and it made the interest rate difference between China and the USA increase, which aggravate China’s pressure to hold the US foreign exchange. Meanwhile, to reduce the influence of international financial crisis and relieve the internal employment pressure, Europe and the United States exert pressure to make RMB to appreciate. Chinese government carried out corresponding measures, People’s Bank of China began to adjust the interest rate from 7.20% on Sep 16th in 2008 down to 5.31% on Dec 23rd in 2009. Meanwhile, Chinese government taken many measures to adjust industry structure, increase internal demand, transfer and absorb the surplus product ability of certain industries. Aiming at the sensitive question of RMB appreciation, Chinese government deepens the exchange rate reform further and People’s Bank of China announced to increase the exchange rate elasticity. Under the situation, the historical exchange rate data is not sufficient to support the changes of future exchange rate and the insufficient data can’t support the general econometric and statistic models. Furthermore, to recover their own economy, every country in the world carries out all kinds of policies
frequently, which increases the uncertainty of exchange rate. The GM (1, 1) model based on grey system theory was proposed by Professor Deng in 1982 and could overcome the weak point that discrete recurrence model couldn’t predict the system in long-term periods. Therefore, Grey System model GM (1, 1) is proposed out to predicted future exchange rate of China in short term.

2. Literature Review. Grey system prediction model GM (1, 1) has been widely used in many areas. Related papers were referred to the areas of securities market prediction, mechanical workout prediction and the improvement of grey prediction methodology. In Cai’s study, the grey prediction is applied to the prediction of stock trends and is found highly effective in the short-term basis (Cai, 2000). Wang (2002) constructed a data mart to reduce the size of stock data and combined fuzzification techniques with the grey theory to develop a fuzzy grey prediction to predict the possible answer immediately (Wang, 2002). Meanwhile, more literatures focused on the application areas of mechanical workout prediction and the wear prediction of engine. To predict the extent of turning force uncertainty quantitatively, Wang, Peng et al. proposed a fuzzy-grey prediction procedure based on the symmetric fuzzy number, linear planning theory and grey set theory (Wang, Peng et al., 2002). And there was also another study on the control and prediction of cutting force (Lian, et al., 2005). Some scholars applied a gray prediction scheme to eliminate the "chattering" disadvantage of the traditional variable structure control (Chou, 2003). Chang, Zhang et al. (2003) calculated the relationship between wear and blow-by gas and the influence of blow-by gas on the Diesel engine performance (Zhang et al., 2003). Furthermore, the unequal interval revised grey model (UIRGM) (1, 1) is presented in Zhang, Li et al research, and they built the model to fit and predict element concentration as determined by oil spectrometric analysis. The results proved that UIRGM (1, 1) determined the exact turning point, and the fitting and prediction results were acceptable (Zhang, Li et al., 2003). In methodology improvement, many scholars made many significant progresses. Under the proposed methodology, the simulation results were shown to be superior to those systems which exploit complicated control variables and rules (Yo-Ping and Chi-Chang, 1996). Tien (2009) proposed a new prediction model called the deterministic grey dynamic model with convolution integral (DGDMC (1, n)) (Tien, 2009).

Based on the works above, it indicates that there are little applications of Grey System model in economic areas. Generally, the exchange rate fluctuates frequently and there is little information because the freedom of observations is limited, so the traditional regression models can’t be applicable to do exchange rate prediction. Therefore, GM (1, 1) model is employed to predict the changes of exchange rate. And Weakening operator method is also applied to improve the prediction accuracy.

3. Methodology and Data.

3.1. Weakening Operator. In this article, Weakening Operator method is applied to improve the prediction accuracy. Therefore, the 1st lag Weakening Operator was drawn in the research to smooth the original data series. And then assume the original sequence is as follows:
Then the sequence treated by Weakening Operator can be given as follows:

\[ X^{(0)}D = (x^{(0)}(1)d, x^{(0)}(2)d, \ldots, x^{(0)}(n)d) \]  

(2)

Where \( x^{(0)}(k)d \) in equation (2) equals to \( \frac{1}{4-k+1}(x^{(0)}(k) + x^{(0)}(k+1) + \cdots + x^{(0)}(n)) \) and \( k = 1, 2, \ldots, n \). When the sequence of \( X \) is monotonic increasing sequence, monotonic decreasing sequence or vibration sequence, \( D \) stands for the weakening operator.

### 3.2. Model GM (1, 1)

Grey system prediction model is employed to predict the fluctuation of exchange rate. Grey prediction discovered and got hold of the system development discipline and quantitatively forecasted system future situation scientifically by handling original data with certain methods and building grey prediction model. GM (1, 1) model is usually used to forecast fluctuation sequence. Based on the qualitative analysis, define appropriate sequence operator, use the sequences dealt with certain operators to build GM (1, 1) model. After accuracy test, it could be utilized to predict further. GM (1, 1) model can be built as four following steps.

1. Making totting-up sequences. Let \( X^{(0)} \) be non-negative sequence and it can be composed as follows:

\[ X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \]  

(3)

Then \( x^{(0)}(k) \geq 0 \) and \( k = 1, 2, \ldots, n \); \( X^{(1)} \) is the 1-AGO sequence of \( X^{(0)} \) which can be expressed as equation (4).

\[ X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)) \]  

(4)

In the equation (4), \( x^{(1)}(k) \) equals to \( \sum_{i=1}^{k} x^{(0)}(i) \), \( k = 1, 2, \ldots, n \). \( Z^{(1)} \) is the proximate mean sequence of \( X^{(1)} \) and it is given as follows:

\[ Z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n)) \]  

(5)

And in equation (5), \( z^{(1)}(k) = 0.5(x^{(1)}(k) + x^{(1)}(k-1)) \) and \( k = 2, 3, \ldots, n \). Furthermore, the following equation is called GM (1, 1) model.

\[ x^{(0)}(k) + az^{(1)}(k) = b \]  

(6)

2. Solving matrix \( Y \) and \( B \), and the parameters sequence. In GM (1, 1) model, the matrix \( Y \) and \( B \) can be given as follows:

\[
Y = \begin{bmatrix}
    x^{(0)}(2) \\
    x^{(0)}(3) \\
    \vdots \\
    x^{(0)}(n)
\end{bmatrix}
\quad \text{and} \quad
B = \begin{bmatrix}
    -z^{(1)}(2) & 1 \\
    -z^{(1)}(3) & 1 \\
    \vdots & \vdots \\
    -z^{(1)}(n) & 1
\end{bmatrix}
\]  

(7)

Then the OLS parameters sequence of GM (1, 1) model can be solved based on the equation (8).
\[ a = \left( B^T B \right)^{-1} B^T Y = \left( \begin{array}{c} a \\ b \end{array} \right) \]  

(8)

(3) Solving whitenization equation. Whitenization equation in GM (1, 1) model is given as follows:

\[ \frac{dx^{(i)}}{dt} + ax^{(i)} = b \]  

(9)

And the time response sequence of GM (1, 1) in

\[ x^{(i)}(k+1) = (x^{(i)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a} \]  

(10)

(4) Model error test. The errors of the model can be given as following sequence:

\[ \varepsilon^{(i)} = x^{(i)}(1) - x^{(i)}(1), x^{(i)}(2) - x^{(i)}(2), \ldots, y^{(i)}(n) - x^{(i)}(n) \]  

(11)

The simulation relative error \( K \) point is as 

\[ \Delta = \frac{1}{n} \sum_{k=1}^{n} \Delta_k, \]  

The mean of simulation relative error is 

\[ \Delta = \frac{1}{n} \sum_{k=1}^{n} \Delta_k, \]  

and the average of accuracy is 

\[ 1 - \Delta. \]

3.3. Data. The sampling data in the application research is chosen from the exchange data from July 2009 to June 2010 and the sequence is made up by monthly data. The data are from People’s Bank of China and the exchange rate of RMB to USD is the middle price of each month. Under the global crisis, the data sequence is in decreasing trend, so the characteristic of the sampling data are applicable to the GM (1, 1) model.


4.1. Data Simulation and Model Test. Based on the original data sequence, the new data sequence of exchange rate of RMB to USD is computed by Weakening Operator method (table 1). Then the original data sequence and the converted data sequence are utilized to predict the further changes of Chinese exchange rate by GM (1, 1) model. Therefore, the basic data in the model is 

\[ x_i^{(0)}(k) \]  

\[ k = 1, 2, \ldots, 12 \]  

. Furthermore, the procedure of grey system software called “main” is utilized to predict the exchange rate changes.

According to Table 2, the stimulation results, the residual errors and the relative errors are given out. The relative errors of the first 10 stimulation value are very small, which are all below 0.1%. And only the relative error of 11st stimulation result is 0.21%, which can be explained by the big exchange rate changes in June 2010. Then the average relative error is 0.06223%, so the prediction accuracy attains to 99.94%. Combining the accuracy of grey prediction model with the reference level table of accuracy test, the prediction accuracy of weakening operator conversion sequence was the 1st level (Table 3).

Combining with the research made by Professor Liu and Deng (2000), \( \alpha \) equals to 0.000348. Therefore, the model GM (1, 1) could do medium-term or long-term prediction. And our destination is to explore the medium-term change tendencies of Chinese exchange rate, so the future 6 months exchange rates are to be predicted.
### Table 1. The original and the converted exchange rate data of RMB to USD

<table>
<thead>
<tr>
<th>Month</th>
<th>Original data</th>
<th>Weakening Operator data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jul-2009</td>
<td>6.8323</td>
<td>6.8251</td>
</tr>
<tr>
<td>Aug-2009</td>
<td>6.8312</td>
<td>6.8245</td>
</tr>
<tr>
<td>Sep-2009</td>
<td>6.8290</td>
<td>6.8238</td>
</tr>
<tr>
<td>Oct-2009</td>
<td>6.8281</td>
<td>6.8232</td>
</tr>
<tr>
<td>Nov-2009</td>
<td>6.8272</td>
<td>6.8226</td>
</tr>
<tr>
<td>Dec-2009</td>
<td>6.8282</td>
<td>6.8219</td>
</tr>
<tr>
<td>Jan-2010</td>
<td>6.8270</td>
<td>6.8209</td>
</tr>
<tr>
<td>Feb-2010</td>
<td>6.8269</td>
<td>6.8197</td>
</tr>
<tr>
<td>Mar-2010</td>
<td>6.8263</td>
<td>6.8179</td>
</tr>
<tr>
<td>Apr-2010</td>
<td>6.8263</td>
<td>6.8151</td>
</tr>
<tr>
<td>May-2010</td>
<td>6.8280</td>
<td>6.8095</td>
</tr>
<tr>
<td>Jun-2010</td>
<td>6.7909</td>
<td>6.7909</td>
</tr>
</tbody>
</table>

Data source: The data is from People’s Bank of China and the Weakening Operator data sequence is computed based on its definition in equation (2).

### Table 2. The stimulation result, residual error and relative error

<table>
<thead>
<tr>
<th>Month</th>
<th>Stimulation Result</th>
<th>Residual Error</th>
<th>Relative Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aug-2009</td>
<td>6.8291</td>
<td>0.0046</td>
<td>0.07%</td>
</tr>
<tr>
<td>Sep-2009</td>
<td>6.8268</td>
<td>0.0030</td>
<td>0.04%</td>
</tr>
<tr>
<td>Oct-2009</td>
<td>6.8244</td>
<td>0.0012</td>
<td>0.02%</td>
</tr>
<tr>
<td>Nov-2009</td>
<td>6.8220</td>
<td>-0.0006</td>
<td>-0.01%</td>
</tr>
<tr>
<td>Dec-2009</td>
<td>6.8196</td>
<td>-0.0023</td>
<td>-0.03%</td>
</tr>
<tr>
<td>Jan-2010</td>
<td>6.8173</td>
<td>-0.0036</td>
<td>-0.05%</td>
</tr>
<tr>
<td>Feb-2010</td>
<td>6.8149</td>
<td>-0.0048</td>
<td>-0.07%</td>
</tr>
<tr>
<td>Mar-2010</td>
<td>6.8125</td>
<td>-0.0054</td>
<td>-0.08%</td>
</tr>
<tr>
<td>Apr-2010</td>
<td>6.8102</td>
<td>-0.0049</td>
<td>-0.07%</td>
</tr>
<tr>
<td>May-2010</td>
<td>6.8078</td>
<td>-0.0017</td>
<td>-0.03%</td>
</tr>
<tr>
<td>Jun-2010</td>
<td>6.8054</td>
<td>0.0145</td>
<td>0.21%</td>
</tr>
</tbody>
</table>

Data source: The data in table 2 is arrange from the predict result of Grey System model.

### Table 3. The reference level table of accuracy test

<table>
<thead>
<tr>
<th>Accuracy Level</th>
<th>Relative Error</th>
<th>Degree of Correlation</th>
<th>Unbiased Variance Ratio</th>
<th>Small Error Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st level</td>
<td>0.01</td>
<td>0.90</td>
<td>0.35</td>
<td>0.95</td>
</tr>
<tr>
<td>2nd level</td>
<td>0.05</td>
<td>0.80</td>
<td>0.50</td>
<td>0.80</td>
</tr>
<tr>
<td>3rd level</td>
<td>0.10</td>
<td>0.70</td>
<td>0.65</td>
<td>0.70</td>
</tr>
<tr>
<td>4th level</td>
<td>0.20</td>
<td>0.60</td>
<td>0.80</td>
<td>0.60</td>
</tr>
</tbody>
</table>

TABLE 4. The predicted results by grey system model GM (1, 1)

<table>
<thead>
<tr>
<th>Month</th>
<th>Jul-2010</th>
<th>Aug-2010</th>
<th>Sep-2010</th>
<th>Oct-2010</th>
<th>Nov-2010</th>
<th>Dec-2010</th>
</tr>
</thead>
</table>

4.2. Exchange Rate Prediction. Grey system software is utilized to analyze the changes of exchange rate with the data in table 1. Firstly, the time response function is computed by

\[ x(k+1) = -19628.602952 \exp(-0.000348*k) + 19635.428052 \]  

(12)

Then the prediction results show that the exchange rate appears to be decreasing trend. Up to Dec 2010, the prediction results are to be respectively as these in table 4.

5. Conclusion. Based on the prediction process and the results, the GM (1, 1) model is applicable to predict the changes of exchange rate which is of fluctuation characteristic and it has more advantages that general econometric and statistic models. Firstly, from the stimulation errors, the relative errors are very small. In the research, it is smaller than 1%. Secondly, from angel of the model accuracy, the prediction accuracy is high and it attains to 99.94%, which makes the accuracy of the GM (1, 1) model get to 1st level. Therefore, employ GM (1, 1) model to explore the change tendency of exchange rate can assure the prediction accuracy. Thirdly, the change trend of exchange rate is in line with the original situation. Finally, the exchange rate is going to go down in the prediction results, which meets the situations nowadays. Therefore, RMB is going to appreciate and it can instruct investors or the government to make macro economy policies.

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